

Fig. 6 (a) The cover in relation to (b) the substrate attached to the substrate carrier

#### IV. CONCLUSIONS

A design approach that is simple in concept, that does not require an expert level of understanding to implement, and that presents a workable solution to the fundamental packaging problem of injecting and extracting energy from a hybrid or monolithic circuit has been presented.

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#### A Note on the Mixed Potential Representation of Electric Fields in Layered Media

M. J. CLOUD, MEMBER, IEEE, AND D. P. NYQUIST,  
MEMBER, IEEE

**Abstract**—A mixed potential formulation is given for electric fields in layered environments. Contributions to the field from charges are identi-

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M. J. Cloud is with Lawrence Technological University, Southfield, MI 48075.

D. P. Nyquist is with the Department of Electrical Engineering, Michigan State University, East Lansing, MI 48824-1226

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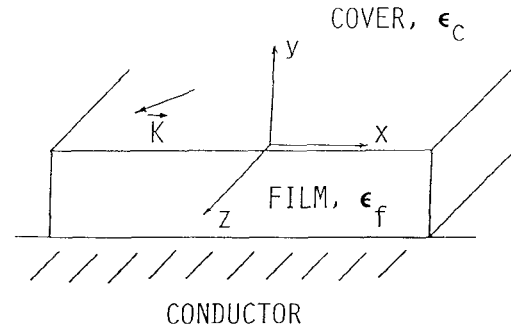


Fig. 1 Configuration of layered media.

fied explicitly through a scalar Green's function for layered media. The outcome is a computationally expedient Sommerfeld integral representation.

#### I. INTRODUCTION

The study of electric fields due to surface currents in millimeter-wave integrated circuits [1], [2] brings to light certain facts about the alternative representation of Hertz potentials. Despite their apparent simplicity, these observations have not appeared previously in this form. The points discussed in this paper bear directly upon the divergent spectral integrations which have been offered on several occasions [3], [4] in the recent literature; it is hoped that ultimately they will find application in avoiding comparatively awkward formulations.

#### II. HERTZ POTENTIAL GREEN'S DYAD FORMULATION

Consider the configuration of layered dielectric media over a conducting half-space as shown in Fig. 1. The electric field  $\vec{E}(\vec{r})$  in the cover, maintained by surface currents embedded in that same layer, decomposes linearly into two parts as  $\vec{E}(\vec{r}) = \vec{E}^p(\vec{r}) + \vec{E}^r(\vec{r})$ . The fields of the right member may be termed the primary and reflected components. A Hertz potential representation of  $\vec{E}$  based upon this decomposition is given by Bagby and Nyquist [1], [2] as follows:

$$\vec{E}(\vec{r}) = (k_c^2 + \nabla \nabla \cdot) \int_S \vec{G}(\vec{r}, \vec{r}') \cdot [\vec{K}(\vec{r}') / j\omega\epsilon] dS' \quad (1)$$

where  $\vec{K}(\vec{r})$  describes source currents on surface  $S$ , and  $\vec{G} = \vec{I} G^p + \vec{G}^r$  is the decomposition of the Hertz potential Green's dyad into primary and reflected components. The dyad scalar components are given as double spectral (Sommerfeld-type) integrals:

$$G^p(\vec{r}, \vec{r}') = \iint_{-\infty}^{\infty} \frac{\exp[j\vec{\lambda} \cdot (\vec{r} - \vec{r}')] \exp[-p_c |y - y'|]}{2(2\pi)^2 p_c} d^2\lambda \quad (2)$$

$$\begin{pmatrix} G_t^r(\vec{r}, \vec{r}') \\ G_n^r(\vec{r}, \vec{r}') \\ G_c^r(\vec{r}, \vec{r}') \end{pmatrix} = \iint_{-\infty}^{\infty} \begin{pmatrix} R_t(\lambda) \\ R_n(\lambda) \\ C(\lambda) \end{pmatrix} \frac{\exp[j\vec{\lambda} \cdot (\vec{r} - \vec{r}')] \exp[-p_c |y - y'|]}{2(2\pi)^2 p_c} d^2\lambda \quad (3)$$

where  $\vec{G}^r = \hat{x} G_t^r \hat{x} + \hat{y} [(\partial G_t^r / \partial x) \hat{x} + G_n^r \hat{y} + (\partial G_c^r / \partial z) \hat{z}] + \hat{z} G_t^r \hat{z}$ . The  $R_t$ ,  $R_n$ , and  $C$  are reflection and coupling coefficients detailed in [1] and [2]. Note that  $p_c = [\lambda^2 - k_c^2]^{1/2}$  is a wavenum-

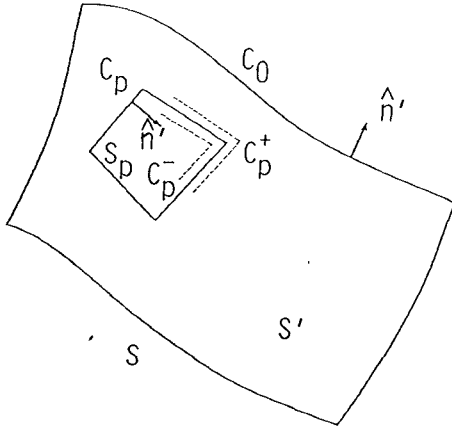


Fig. 2. Geometry of surface over which currents exist.

ber parameter,  $\vec{\lambda} = \hat{x}\lambda_x + \hat{z}\lambda_z$  is a vector spatial frequency, and  $d^2\lambda = d\lambda_x d\lambda_z$ .

The first term (with  $k_z^2$  factor) in  $\vec{E}(\vec{r})$  is proportional to the vector Hertz potential contribution to  $\vec{E}$  (produced directly by currents  $\vec{K}$ ), while the second term is subsequently treated as the negative gradient of the scalar potential. In the present formulation, and in many similar ones [3], [4], it is found that boldly exchanging the order of differentiation (i.e., the  $\nabla\nabla\cdot$  operation) with integration too many times leads to convergence problems with the resulting inner-nested spectral integrals. Care will be taken in the present development to allow only one such interchange; moreover, the impact of the derivative on the Green's function is subsequently minimized by a technique of integration by parts.

Fig. 2 illustrates the geometry of surface  $S$ , supporting currents of integrated electronics. Assume that  $\vec{K}(\vec{r})$  is a continuous function everywhere on  $S$ , except possibly at the boundary  $C_p$  of subregion  $S_p$ . The exception is made because, although one physically expects continuous currents, discontinuities could be introduced mathematically by subsectional basis functions in numerical solutions.  $C_p^+$  and  $C_p^-$  are auxiliary boundary contours just outside and inside of  $C_p$ . Take  $C_0$  to be the outer contour of  $S$ . Let  $S' = S - S_p$ , with  $\hat{n}'$  the outward normal unit vector to  $S'$ . It is important to note here that the principal Green's function  $G^p$  may in fact be represented either in Sommerfeld integral form or simply as  $\exp(-jkR)/4\pi R$ ; the latter form points out explicitly the nature of its source-point ( $R = |\vec{r} - \vec{r}'| = 0$ ) singularity. This singularity, unless handled properly, invalidates a subsequent application of the divergence theorem. To maintain complete rigor a small area should be excluded from  $S$ , at the location  $\vec{r}$ , to preclude the coincidence of  $\vec{r}$  and  $\vec{r}'$ , during the spatial integration in (1). However, at points where  $\vec{K}$  is continuous there is no contribution from either the excluded areas or its boundary contour. Therefore, the exclusion process is not detailed here. Points of surface current discontinuity are handled in the following section.

### III. PRIMARY COMPONENT OF ELECTRIC FIELD

Consider first the primary component of  $\vec{E}$ . The electric scalar potential contribution to  $\vec{E}^p$  is  $-\nabla\Phi^p$ , where

$$\begin{aligned}\Phi^p &= -(1/j\omega\epsilon)\nabla\cdot\int_S G^p\vec{K}dS' \\ &= -(1/j\omega\epsilon)\left[\int_{S'}\nabla\cdot(G^p\vec{K})dS' + \int_{S_p}\nabla\cdot(G^p\vec{K})dS'\right].\end{aligned}\quad (4)$$

This expression is subsequently cast into a form which explicitly exposes its electric charge sources. Although the method is conventional, it is included here to guide the identification of similar contributions to the reflected potential. The procedure is less obvious in the latter case.

Exchanging the divergence and spatial integration prompts vector manipulation of the integrand as follows:

$$\nabla\cdot(G^p\vec{K}) = G^p\nabla\cdot\vec{K} + \nabla G^p\cdot\vec{K} = -\nabla'\cdot(G^p\vec{K}) + G^p\nabla'\cdot\vec{K}. \quad (5)$$

Into the last term a substitution from the surface continuity equation,  $\nabla\cdot\vec{K} = -j\omega\sigma$ , is made. The scalar potential becomes

$$\begin{aligned}\Phi^p &= (1/j\omega\epsilon)\left[\int_{S'}\nabla'\cdot(G^p\vec{K})dS' + \int_{S_p}\nabla'\cdot(G^p\vec{K})dS'\right] \\ &\quad + (1/\epsilon)\int_S(G^p\sigma)dS'\end{aligned}\quad (6)$$

to which the 2-D divergence theorem is applied to give

$$\begin{aligned}\Phi^p &= (1/\epsilon)\left[(1/j\omega)\oint_{C_0} G^p(\hat{n}'\cdot\vec{K})dl' \right. \\ &\quad \left. + (1/j\omega)\oint_{C_p} G^p\left[\hat{n}'\cdot(\vec{K}^+ - \vec{K}^-)\right]dl' + \int_S G^p\sigma dS'\right] \\ &= (1/\epsilon)\left[\oint_{C_0} G^p\rho_{10}dl' + \oint_{C_p} G^p\rho_{1p}dl' + \int_S G^p\sigma dS'\right]\end{aligned}\quad (7)$$

where  $\vec{K}^+$  and  $\vec{K}^-$  are values of surface current on  $C_p^+$  and  $C_p^-$ , respectively. Thus  $\Phi^p$  is written in terms of line and surface charges, while the original derivatives have been effectively removed from  $G^p$ .

### IV. REFLECTED COMPONENT

For the reflected potential,

$$\Phi^r = (-1/j\omega\epsilon)\nabla\cdot\int_S \vec{G}^r\cdot\vec{K}dS' \quad (8)$$

one may follow the same general sequence of steps as above, but this time exploiting the specific form of  $\vec{G}^r$ . Following the procedure beginning with (5), and defining a scalar Green's function  $G^r = G'_x + \partial G'_y/\partial y$ ,

$$\begin{aligned}\nabla\cdot(\vec{G}^r\cdot\vec{K}) &= \nabla\cdot\left\{\hat{x}G'_xK_x + \hat{y}\left[(\partial G'_x/\partial x)K_x + (\partial G'_y/\partial z)K_z\right] + \hat{z}G'_yK_z\right\} \\ &= (\partial G'_x/\partial x)K_x + (\partial G'_y/\partial z)K_z \\ &\quad + (\partial/\partial y)\left[(\partial G'_x/\partial x)K_x + (\partial G'_y/\partial z)K_z\right] \\ &= (\nabla_{xz}G'_x)\cdot\vec{K} + [\nabla_{xz}(\partial G'_y/\partial y)]\cdot\vec{K} \\ &= (\nabla G^r)\cdot\vec{K} = (-\nabla'G^r)\cdot\vec{K}.\end{aligned}\quad (9)$$

Carrying the analogous steps to completion, one finally arrives at

$$\Phi^r = (1/\epsilon)\left[\oint_{C_0} G^r\rho_{10}dl' + \oint_{C_p} G^r\rho_{1p}dl' + \oint_S G^r\sigma dS'\right]. \quad (10)$$

Therefore the reflected scalar potential is expressible in terms of effective charges weighted by an appropriate Green's function. It is noted that a convenient form of the scalar reflected Green's function arose from the specific form of the Hertz potential Green's dyad cited earlier. Since the conversion process has effectively removed derivatives from this function, the new formulation is expected to be computationally efficient. Finally, the

$\vec{E}$  field in the cover region is expressed as

$$\vec{E} = -j\omega\mu \int_S \vec{G} \cdot \vec{K} dS - \nabla(1/\epsilon) \cdot \left[ \oint_{C_0} G\rho_{10} dl' + \oint_{C_p} G\rho_{1p} dl' + \int_S G\sigma dS' \right] \quad (11)$$

where  $G = G^p + G^r$  is a Sommerfeld integral representation of the scalar potential Green's function for layered media. The gradient operator may (at interior points) be exchanged with the spectral integrals without rendering those Sommerfeld integrals nonconvergent.

## V. SUMMARY AND CONCLUSIONS

The preceding sections illustrate a procedure for avoiding the needless imposition of derivatives onto the Green's function, thus avoiding convergence problems. In the process of converting to a less singular formulation, unknown surface and line charges are introduced explicitly into the problem. Presumably, for many applied problems, these charge functions could be expanded in suitable moment method basis sets along with the original surface currents.

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## Dominance of Resistive Losses over Hysteretic Losses in Ferromagnetic Conductors

K. K. THORNBER

**Abstract**—In the presence of ferromagnetic conductors, electromagnetic signal propagation might be seriously degraded through hysteretic losses. It is shown, however, that, independent of frequency, such losses are always smaller than the ordinary resistive losses one would calculate for a similarly resistive, equivalent paramagnetic material. This indicates that measurements of conductivity and permeability suffice to bound hysteretic losses, obviating the necessity for measurements of hysteresis at operating frequencies.

## I. INTRODUCTION

Electrical interconnection within novel, multichip packaging technologies [1] can require magnetic materials in order to satisfy the constraints of processing. Thus, for example, nickel can be used as a vapor barrier protecting an underlying copper layer against the products and reactants present during the curing of an adjacent polyimide insulating layer. Being capable of electrodeless deposition, Ni can also be used for thick, vertical structures

such as vias. Other magnetic materials might also be included to meet special objectives.

Whenever magnetic materials are used in an electromagnetic environment, hysteretic losses are induced by the time-dependent fields [2]. Such losses are difficult to measure and to calculate. For example, a calculation requires knowledge of  $B(x, t)$  given  $H(x, t')$  for  $t' < t$ , under spatially nonuniform conditions, i.e., field strengths and hence hysteresis that are different in each portion of the material. Boundary conditions introduce additional complications.

It is the purpose of this short note to observe that under rather general conditions, the actual power losses to magnetic hysteresis in conducting materials are less than power losses to electrical resistance, the latter being calculated for a situation in which locally  $B(x, \omega) = \mu_e(\omega)H(x, \omega)$  and  $J(x, \omega) = \sigma(\omega)E(x, \omega)$ . For the purposes of the calculation of the resistive losses, the material is conductive and either paramagnetic or diamagnetic. Since the conductivity,  $\sigma$ , and the permeability,  $\mu_e$ , are readily measurable, this approach for bounding hysteretic losses can be implemented directly.

## II. DERIVATION

That resistive losses exceed hysteretic losses is derived as follows. Starting with Ampere's law at the frequency of interest (ignoring the displacement currents),

$$\nabla \times H = \sigma E \quad (1)$$

and taking the direction of penetration into the magnetic material to be  $\hat{x}$ , it follows that the largest transverse components of the fields are related by

$$\sigma E_z(x) = \delta^{-1} H_y(x) \quad (2)$$

where  $\delta \equiv (-i\omega\mu_e\sigma)^{-1/2}$ , the complex skin depth. The skin depth can be obtained by combining Gauss' law,

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (3)$$

with (1) to yield

$$\nabla^2 H = \frac{\sigma \partial B}{\partial t} = -i\omega\sigma\mu_e H. \quad (4)$$

In each volume element, therefore, the resistive power dissipation  $\sigma|E|^2/2$  becomes

$$P_E(x) = \frac{\sigma|E|^2}{2} = \frac{|H|^2}{(2|\delta|^2\sigma)} = \pi\nu\mu_e|H|^2 \quad (5)$$

where  $\nu = \omega/2\pi$  is the frequency of interest.  $P_E$  is thus the local rate of energy dissipation per unit volume which would be calculated for a paramagnetic material of effective permeability  $\mu_e$ .

The actual hysteretic loss at  $x$  per unit volume is given by

$$P_H(x) = \nu \oint H(x) \cdot dB(x) \quad (6)$$

which under saturation conditions can be expressed in terms of  $H_c$ ,  $H_s$ , and  $B_s$  as defined in Fig. 1 as

$$P_H(x) = 4\nu H_c(x) B_s(x). \quad (7)$$